P425/1
PURE MATHEMATICS
Paper 1
July/August 2017
3 hours

WAKISSHA JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer* **all** *the eight questions in section* A *and any* **five** *questions from section* B.
- *Any additional question(s) answered will* **not** *be marked.*
- *Show* **all** *necessary working clearly.*
- *Begin each answer on a fresh page of paper.*
- *Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A (40 MARKS)

Answer all questions in this section

SECTION A

1. Solve for x the equation $3\log_8 x + 2\log_x 8 = 5$. (5 marks)

2. Evaluate $\int_0^{\pi/4} \sec^4 x \, dx$. (5 marks)

3. Use Maclaurin's theorem to show that the expansion of $e^{-x}\sin x$ up to the term in x^3 is given by $\frac{x}{3}(x^2 - 3x + 3)$. Hence evaluate $e^{-\pi/3}\sin\frac{\pi}{3}$ correct to four decimal places. (5 marks)

4. Find the values of x lying between -180° and 180° that satisfy the equation $10\sin^2 x + 10\sin x\cos x = \cos^2 x + 2$. (5 marks)

5. If the equation $x^2 + ax + p = 0$ and $cx^2 + 2ax - 3p = 0$ have a common root, show that $p(c+3)^2 = 5a^2(c-2)$. (5 marks)

6. A line P passing through the point Q(1, 2, -1) is perpendicular to the plane $x + 2y + 3z = 14$. Find the Cartesian equation of P. (5 marks)

7. If $y = \frac{\sin x}{1+\cos x}$, prove that $\frac{dy}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$ (5 marks)

8. Sketch the graph of a parabola whose parametric coordinates are $(3t^2 - 2, -6t)$. Show clearly the focus and the directrix. (5 marks)

SECTION B (60 MARKS)

Answer any five questions from this section.

9. (a) Prove that $\frac{\cos 3x}{\cos x} - \frac{\cos 6x}{\cos 2x} = 2(\cos 2x - \cos 4x)$. (4 marks)

   (b) Show that $\tan^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \frac{\pi}{4}$ (8 marks)

10. Show that for real x, the function $f(x) = \frac{x^2 - x - 6}{x - 1}$ can take all real values.

    Hence sketch the curve of $f(x)$. (12 marks)

1. (a) Express $\frac{(2-i)^2(3i-1)}{i+3}$ in Modulus-argument form. (5 marks)

(b) Prove that if $\frac{(z-6i)}{(2z-1)}$ is purely imaginary, then the locus of the point representing z in the Argand diagram is a circle. Hence find the center and radius of the circle. (7 marks)

12. (a) Solve the equation $ye^{y^2}\frac{dy}{dx} - e^{-x} = 0$. (4 marks)

(b) John walks towards a trading center which is 1,000m away at a rate which is proportional to the distance he still has to cover. He starts by walking at a speed of $1ms^{-1}$ from his home towards the trading center. How many minutes does he take to cover 600m from his home?

(8 marks)

13. (a) The equation of an ellipse is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b > 0$.

A tangent drawn to the upper of the ellipse at (m, n) cuts the x-axis at a point of a distance c from the origin. Show that $\frac{a^2}{b^2} = \frac{mc - m^2}{n^2}$. (6 marks)

(b) The normal to the curve $xy = 4$ at the point $p(2p, \frac{2}{p})$ meets the curve again at point Q. Find the coordinates of point Q. (6 marks)

14. (a) Evaluate $\int_0^2 5x\sqrt{(1+x^2)}\, dx$ (5 marks)

(b) Find the volume generated by rotating through 360^0 about the x axis, the area in the first quadrant enclosed by the curve, y-axis and the line $y = 2$.

(7 marks)

15. (a) The position vectors of points A, B and C are $\frac{1}{4}(a + 3b)$, $\frac{1}{2}(3a - b)$ and $\frac{1}{8}(3a + 5b)$ respectively. Prove that the points lie on a straight line and determine the ratio AB : BC. (6 marks)

(b) The distance of the point A(4, -1, 2) from a plane is $\sqrt{3}$. Given that the vector $i + j + k$ is perpendicular to the plane, find the Cartesian equation of the plane. (6 marks)

16. Use the remainder theorem to factorise $x^4 + 3x^2 - 4$ completely and hence express $\frac{2x^3 - x^2 - 7x - 14}{x^4 + 3x^2 - 4}$ in partial fractions. Hence or otherwise find $\int \frac{2x^3 - x^2 - 7x - 14}{x^4 + 3x^2 - 4} dx$.

(12 marks)

END